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ABSTRACT

This report provides a detailed analysis of skill complexes in the multi-year sequence of arithmetic and general mathematics that are covered in school textbooks prior to the beginning of formal algebra. The analysis is organized to show the shape of the learning opportunity that most students are likely to get in school by about the time they reach the end of grades 1-8. The focus is on the mathematical tasks that students do, and in particular what they do today so that improvements in learning opportunities can be designed. Students mainly get opportunities to learn how to do things in mathematics, rather than to understand mathematical meaning. Similarities in content across textbooks for the same grade level are far more striking than the differences, but there are real differences in the way many topics are presented, and in the sequencing of topics during the year. There seems to be much consensus regarding what schools can, or at least do, accomplish. The report then briefly highlights the follwoing topics: whole numbers, computation with whole numbers, common fractions, decimals, percent, ratio and proportion, probability and statistics, integers, algebra, geometry, measurement, and special topics, each with subtopics. (MNS)

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The report provides a detailed analysis of skill complexes in arithmetic and general mathematics that are covered in school textbooks prior to the beginning of formal algebra. The analysis is organized to show the shape of the learning opportunity that most students are likely to get in school by about the time they reach the end of Grades 1-8.

THE SHAPE OF LEARNING OPPORTUNITY IN SCHOOL ARITHMETIC AND GENERAL MATHEMATICS

Aaron Buchanan

The educational leadership in this country is still trying to improve on a system for teaching and learning whose shape it does not clearly see. In the 1960s, "school reform" was intended to replace a lot of things schools were doing then. Today, we have an "effective schools" movement aimed at improving on what schools are doing now. Over the years, the scope of change that "leadership" is trying to bring about is different, but one thing, at least, is not: with all the talk about what and how schools ought to teach, it has been hard to find much discussion, then or now, of what schools do. Leadership in education has become equated with change. To spend much time explaining current schooling practices--what is taught and basically how it's done--seems too much like trying to maintain the status quo. Research, these days, is preoccupied with the amount of "engaged time" students spend in learning academic tasks, but it hasn't shed much light on the academics students are regularly "engaged" in or how "tasks" are organized into broad learning opportunities that often extend across several years of schooling.

What do schools do? More to the point: if students were to keep up with the learning opportunity that schools provide what would they know and know how to do as they passed different transition points on their way to becoming adults? The answers are important if we are to have informed and effective leadership and, more important, if efforts to promote change in education are to lead anywhere. The years between

1960 and 1980 have had a chastening effect, especially on educational research. After an enormous amount of effort to reshape schools, one of the most striking results has been the discovery that most of what schools do must be fairly stable. Most changes evolve slowly, especially ones that affect school organization and the way teachers spend time available for instruction. In order to bring about successful improvement of schooling, you have to know how many degrees of freedom there are to work with and where they exist.

Nowhere has there been more misunderstanding over what schools do and where there is enough margin for change than in mathematics. "New mathematics" programs have basically been written off by the public as unsuccessful. Reasons that are given vary, but one that is as close to the truth as any is that too much change was attempted over too short a time. In fact, "new mathematics" has been anything but a failure. Even in a time of "back to basics," mathematics teaching in U.S. schools is different than it was thirty years ago, but, to see the difference one has to recognize what schools are doing now compared with what they were doing then. Merely attending to the "talk" of what schools are supposed to do now or what they were supposed to do in the past reveals very little. The differences exist in practices, not goals and objectives. Now, as then, schools in the aggregate are set up to provide some fairly broad opportunities for learning different collections of very specific things. For elementary and secondary schools, there is one fairly continuous learning opportunity that extends through the elementary grades to the beginning of algebra, and a series of shorter, one-year opportunities after that. The first one is the most interesting, not

because it's so long, but because there are no natural breaking points along the way. School arithmetic and general mathematics is one multi-year sequence for developing a large number of interrelated skills and concepts. Trying to identify what schools teach by the end of a particular grade level is bound to be imprecise, at best. Most of the skill development that begins with whole numbers at grade 1 is not very complete before grade 5, and work with fractions and decimals, which often begins in grade 2, isn't completed until students get to ratios and proportions just prior to beginning algebra.

Learning Opportunity and the Use of Textbooks

What follows is an analysis of arithmetic and general mathematics instruction that schools currently provide. It deals with the question: "What are U.S. schools teaching?" or more accurately, "What is the shape of learning opportunity that schools in the aggregate provide?" Clearly these questions can have several quite different yet quite legitimate answers depending on the data one examines. The source data for the present analysis are school textbooks. On first impression, this analytic approach may seem indirect, but research on school practices indicates that the learning opportunity that authors, editors, and publishers provide in textbooks and the learning activities that teachers, administrators, and supervisors provide in schools correspond closely.

We will not take sides on whether teachers use textbooks well or poorly or whether textbook publishers are too conservative or too innovative. It is enough to simply acknowledge that textbooks exist for schools to use, that teachers in schools do use them with day-to-day regularity when

they teach mathematics to students, and that the performances of students that we are able to observe most clearly and to explain best are the ones lying closest to the learning opportunity that textbooks provide. With these qualifications well out in front of our inquiry we can proceed to examine textbooks as a fairly rich data source on the learning opportunities that schools provide students prior to a first course, if any, in algebra.

Our long range motives for observing learning opportunity in this way are straightforward. We want a better understanding of what to expect and look for in students' performances at the culmination of their first and most comprehensive and continuous learning experience in mathematics. For many students, arithmetic and general mathematics is the only mathematics they will ever have (or want to have, an opportunity to learn. By knowing more of what we can expect to see in student performances now, schools and research are in a better position to explain why observed performances are too often something less than we would like--and they are also in better shape to do something about it.

Presently, schools are forced to keep a double set of books in order to account for the performances of students. One set accounts for what schools would like to see students learn. The other accounts for what students have learned. With the rise of legislated "accountability," the emphasis has shifted from one set of books to the other--away from the learning opportunities schools aspire to provide and toward the performances they are more certain they can deliver. So far, most everybody sees the difference. The risk will come if statements of minimal competencies, which exist everywhere, come to be regarded as the practical boundaries for learning opportunities that schools expect and ought to be expected to

provide. The trick is to effect a match of the second set of expectations with the first and not the other way around. To do this requires information regarding the learning opportunity that characterizes school instruction now. Without a clear view of learning opportunity and its relationship to student performance, schools--and school research-- have greatly diminished capacity to provide the additional learning opportunity needed by a current generation of students and to design improvements in learning opportunity for future generations coming up along the line from the early elementary grades.

How Textbooks Organize Learning Opportunity by Topic

Textbooks are organized by topics, such as "adding fractions with unlike denominators" or "finding averages." Although expressed as mathematics themes, these topics almost always concentrate on some aspect of student performance. Even with topics such as "parallel and perpendicular lines" the emphasis is on identifying these relationships in geometric figures and diagrams. To put it another way, students mainly get opportunities to learn how to do things. (Considerably less emphasis is placed on what an "average" is than on how you find it.)

Partly, the heavy emphasis on performance reflects a fairly common image of conventional characteristics of the regular classroom. You have one teacher, 25 to 30 students, something to be taught and learned, and about 30 to 50 minutes of time to do it. Some time will obviously be used by the teacher in providing information, explaining why, and often demonstrating how to a large group of students. Most of the time will be spent by students working on instructional tasks on their own. Therefore, authors

and editors capture as much mathematics as possible in paper and pencil tasks. Even when students are responding to direct questions, the answers require specific and often fairly complex performances.

The emphasis on performance also reflects a preoccupation over two decades on observable objectives. Nowhere has the enormous effort to describe objectives "in behavioral terms" had more impact than in mathematics. As a consequence, relatively little of the learning opportunity in textbooks involves things that students are supposed to know; almost all of it is taken up with things students are supposed to know how to do.

Topics usually deal with parts of performance. Performance such as computation and its application in social and cultural situations is taught over a period of several years. Within any single year, topics of instruction deal directly with significant parts of such performance. These topics tend to arrange themselves linearly according to how performance has been disassembled into parts that build on one another. Computation algorithms are never taught as a single performance and so are never organized within a single topic. Even the simplest algorithms tend to spread over the first and last half of a school year and usually over two years and more.

Teachers can, of course, go beyond the substance of the text. Most textbooks are filled with marginal comments to teachers telling how to embed student performances in the larger context of mathematical structures and their elated concepts and skills. But the authors and editors are actually able to capture very little of this on paper, and it is unreason-able to expect that teachers will go very far beyond a learning opportunity that deals with how and when to perform.

Learning Opportunity Across Different Textbook Series

It's quite appropriate and accurate to talk about "a" learning opportunity as such in mathematics instruction up through about grade 8. Only a half dozen or so textbook series account for what the vast majority of what schools buy and classroom teachers use. Furthermore, different textbook series look quite a lot alike. Things that students can be expected to know or know how to do are about the same from one series to another. Together, the similarities that occur across textbooks for the same grade level are far more striking than the differences. By the end of the year, students will have seen, heard, and practiced about the same things as a direct result of what's covered in the textbook regardless of the textbook series.

Why do textbooks look so much alike? The learning opportunity provided in an individual mathematics textbook is not determined solely at the author's or publisher's discretion. More realistically, it is shaped by an interacting set of "forces." The topics covered at a particular grade level are determined to some extent by regulations imposed through state law, especially in larger states which have state-wide textbook adoptions, by the limited number of topics that can realistically be included in a year's worth of instruction, by the per-pupil costs that schools are willing to pay for teaching materials, and by economic competition which has the effect of limiting instances where a learning opportunity provided by one publishing company gives them an appreciable marketing advantage over others.

It would be a mistake to look at learning opportunity with the notion that schools are captive consumers of what publishers choose to provide. It would also be a mistake to think that regulations imposed by schools through their state and local education agencies take away all of the leeway publishers have in what they can offer. The truth lies somewhere in between. Two different textbooks for the same grade level cover about the same topics. That much is fact. The unique topics in any textbook series account for a very small fraction of instruction in a school year. Publishers have tended to copy each other's practices, especially ones that seem to be successful in selling textbooks. We've now come to a point where the same topics are covered to about the same degree by the end of each grade level regardless of whose textbook you buy. Schools, on the other hand, may not have regulated all of the creative differences out of textbooks, but they have developed grade-by-grade lists of "must" topics that are so long that instruction at any grade level would take more than a year to cover all of them.

Different textbooks series do give different treatments to some things. The topics that would account for all of the prototypical school year may be essentially the same, but there are real differences in the way they are presented. For example, some publishers treat the standard task of finding what percent one number is of another as a variation on the (by now) more familiar task of finding an equivalent fraction with a specific denominator, in this case 100. Other publishers treat the task as a proportion problem which can be solved by cross multiplication. Textbooks for the same grade level also differ greatly in the way they sequence topics during the year, although the topics they cover do seem to represent about the same plateaus in what students are supposed to

be able to do by the end of the year. Some textbooks at grades 7 and 8 re-present the algorithms for multiplying and dividing with fractions at the same time that they review algorithms for multiplying and dividing whole numbers. Others tend to keep all of the topics dealing with decimal computation separate.

Learning Opportunity as Potential Instruction

Teachers may not cover all of the topics that are provided in textbooks, but they are unlikely to cover more. Teachers do use materials that are not in their regular mathematics textbook, but these materials usually involve supplementary activities that provide intensive practice on topics which teachers have already used their regular textbook to introduce. Therefore, the learning opportunity proposed by textbooks probably represents an upper bound in the topics that will actually be covered in most all of the active instruction in classrooms.

Individual teachers may create their own options, of course. Some may be able to take great advantage of small differences in what one textbook series provides and another one doesn't. Still, the salient impression one gets from analysis of the different textbook series in mathematics is that the learning opportunity they provide functions about the same way as a single program of national priorities. No one close to policy making in mathematics education would like to admit it, but, although there may be great diversity of opinion in what students should know or know how to do by the end of each grade level, there seems to be much clearer consensus regarding what schools can, or at least do, accomplish.

The school community, the academic community, and often the publishing community all find it convenient to think that textbooks are used mainly to support some larger program with a more visible structure than what textbooks have themselves. In practice, mathematics textbooks function less like a passive resource and more like an active determinant of the mathematics program. There is too much detail and sequence in the activities that textbooks provide for us to think that teachers don't use them consistently in classrooms. The relationship between the structure of a mathematics textbook and some of the most basic needs that schools have in providing day-to-day classroom teaching is both direct and clear. First, the content of textbooks is particular rather than general and specific rather than broad. The most tangible unit of structure is an individual lesson intended for somewhere between 30 and 50 minutes of the teaching day. Most lessons provide some sort of explanation, a few guided practice problems, and quite a lot of independent practice for students to do mainly on their own. Second, the content of textbooks is sequential. There is at least an implicit intention that lessons be followed in the sequence in which they occur, although some selection and resequencing of an individual lesson or small clusters of lessons by teachers is not precluded. Third, the number of lessons dealing with a common topic is small. The overall sequence of lessons in a prototypical year changes topics every three or four days, especially at intermediate and upper grade levels. This kind of pacing is justified as a catalyst to student motivation, but it also assumes that textbooks are used by teachers mainly from front to back as a lead-in to active instruction, not as passive support. Finally, textbooks

are complete. If lessons are taught by teachers more or less in sequence beginning in the fall, then the textbook for each grade level will take about one school year to get through. The learning opportunity in textbooks is "potential" instruction, of course, because it hasn't happened yet. Classroom teachers are seldom under any mandate to teach the textbook just as it is, and it's fairly common for them to fall at least one or two units or chapters short of completing the entire textbook by the end of the year. Besides, it's not at all uncommon for teachers to skip some lessons or whole topics that seem to be too hard.

The "Shape" of Learning Opportunity

The shape of learning opportunity in general mathematics extends from kindergarten through grade 8. Textbooks show no natural breaks at grade 6 or any other intermediate grade level. Schools establish some administrative breaking points usually by creating middle schools that cluster grades 5 or 6 through grade 8 or junior high schools that cluster grades 7-9. Textbook series don't show sharp transitions in what is taught across these clusterings, although the look of textbooks for grades 7 and 8 is usually different from what is published for earlier grades.

An analysis of learning opportunity in mathematics that only goes to grade 6 is incomplete. One must look at what's yet to come in grades 7 and 8 in order to fill out the corners of where potential instruction has been designed to move all through elementary school. There are caveats, however. A great many students take their first formal course in algebra in grade 8. If ~~they~~ have completed all of the topics covered

in grades 6 and 7, they won't miss much that's new in grade 8. There is an introduction to integers, including computation with integers. That's mostly new. There are usually a dozen or so lessons in algebra that mainly get through the solution of first order equations in one variable containing both addition/subtraction and multiplication/division (e.g., $3x + 2x + 4 = 29$). Those are new too. But both topics are the main part of the introduction to algebra that students who move out of the general mathematics sequence at the end of grade 7 will cover during the first semester of algebra in grade 8.

The shape of learning opportunity in mathematics prior to a formal first course in algebra does not complete its form until grades 6, 7, and 8. However, in looking at what textbooks provide in grades 6, 7, and 8, there isn't much from earlier grades that doesn't show here in one way or another. The shape of learning opportunity is so linear that only the most elementary work done in the early grades with simple geometric shapes, basic measurement with ruler units, money denominations, time, and simple counting and computation facts is completed before grades 6, 7, and 8.

What one sees from the vantage point of looking down (or back) from grades 7 and 8 is several skill complexes representing accumulations of learning opportunities provided in a few general areas of mathematics such as whole numbers, computation with whole numbers, common fractions, decimals, geometry, measurement, and so on. Some

topics in mathematics represent a learning opportunity that is about as complete as regular schooling will make it. These areas include:

whole numbers

computation with whole numbers

common fractions

decimals

percent

In other areas the learning opportunity is fairly incomplete. This happens because topics represent an introduction that will be picked up in other, more formal mathematics courses at higher grade levels. These areas include:

integers

ratio and proportion

probability and statistics

algebra

special topics (mainly number theory)

A lot of incomplete learning opportunity also occurs in measurement and geometry, but here things are simply fragmented. Some measurement topics dealing with length, money, time, and perhaps liquid capacity are fairly complete. Other topics involving mass (weight), volume, and temperature can't be taken very far unless teachers supplement activities provided in a textbook with a lot of laboratory work. Otherwise, about all that can be accomplished is to learn how to use tables of equivalent units and to remember appropriate units and sensible measures for some standard things like body temperature, weight of a car,

and height of a house. With geometry, learning opportunity is badly fragmented by indecision over what schools should be teaching. Beyond the recognition of some geometric figures like circles and basic polygons and the recognition of some basic relationships like "parallel" and "perpendicular," the learning opportunity amounts to what we once would have called "exposure" to a wide variety of different topics. Little is presented other than to show briefly how some things work. Not much is expected in the way of student performance. Furthermore, many of the topics under geometry (there are more topics in geometry than anywhere else) come from grades 7 and 8 where they are supposed to be optional for all but the most able students.

Once again, the reader should view this analysis as a description of what schools in the aggregate are potentially in position to teach now. The shape of learning opportunity shown here should not be viewed as representing all that schools should attempt to teach or all they are likely to be able to teach in the future. Starting with an analysis of "what is" now, we have a sound technical basis for considering how to improve things in the future.

WHOLE NUMBERS

Counting

Determine how many things are in a set.

Continue a counting sequence by ones, fives, tens, or hundreds.

Identify positions in an ordered set of things (e.g., 27th).

Place value and number names

Give standard arabic numerals for written word names.

"Say" word names for numbers.

Identify place values and periods (hundreds, thousands, millions, and billions) in a standard numeral.

Order, comparison, and rounding

Compare two numbers using < and > symbols.

Place three or more numbers in order from least to greatest (smallest to largest) and greatest to least.

Round numbers to the nearest specified place value (and especially to the largest place value--represented by the first digit).

COMPUTATION WITH WHOLE NUMBERS

Computation facts

Recall "facts" through nines.

Computation algorithms

Add two or more numbers (up to five or six) with any number of digits in them (up to about six).

Subtract two numbers with any number of digits (up to about six).

Multiply and divide by any number (up to three or four digits). Be able to give remainders in division as whole numbers, a fraction of the divisor (preferrably in lowest terms), and a decimal (by "adding" zeros to the dividend if necessary).

Standard word problems*

Use the appropriate operation(s) to answer questions in standard word problems which:

A. Combine two quantities ("How many together?").

B. Remove a smaller quantity from a larger one ("How many are left?").

C. Compare two quantities ("How much larger or smaller is one than the other?").

D. Combine several quantities of the same size ("How many altogether?").

E. Separate a larger quantity into parts of a certain size ("How many parts are there?").

*Use of computation with other specific applications are covered in Measurement and Percent.

COMPUTATION WITH WHOLE NUMBERS (continued)

Standard word problems (continued)

F. Separate a larger quantity into a certain number of parts ("How many are in each part?").

Estimation

Make reasonable estimates of sums, differences, products, and quotients usually by computing with numbers rounded to their largest place value.

COMMON FRACTIONS

Part/whole relationships

Use common factions, mixed numbers, and whole numbers to express relationships between parts of a region (or a line segment) or parts of a set, and all of it. This includes the recognition of a unit region or regions divided into parts that are equal in size and a unit set or sets whose parts are equal according to how they are counted regardless of their "size." Identify the "numerator" and "denominator" of a fraction.

Equivalent fractions

Extend or reduce fractions to higher and lower terms. Express common fractions as mixed numbers and whole numbers (and vice versa).

Express the same part/whole relationship using two equivalent fractions. Generate a set of fractions that are equivalent. Recognize that multiplication of the numerator and denominator by the same number (except 0) will always generate an equivalent fraction but adding or subtracting the same number will not.

Compare common fractions, common fractions and mixed or whole numbers, mixed numbers, and mixed numbers and whole numbers using such terms as more, less, greater, smaller, the same, equal, or not equal, and the symbols <, >, =, ≠, ≯, and so on.

Identify the least common denominator for two fractions from the prime factorization of their denominators. Identify the greatest common divisor, from the prime factorization of the fraction's numerator and denominator.

COMMON FRACTIONS (continued)

Computation

Add, subtract, multiply, and divide with common fractions, common fractions and mixed numbers, and mixed numbers and whole numbers. (Denominators are usually "nice" ones such as 2, 4, 6, 8, 12, 16, 18, 24, 32, and 64.) Preferred answers are "simplified" as far as possible which means reducing proper fractions to lowest terms and changing improper fractions to mixed numbers (in lowest terms) or whole numbers.

Express division of two numbers as a fraction (15 ÷ 5 and $\frac{15}{5}$ are the same number) and vice versa.

Reciprocal

Identify the reciprocal of a common fraction, mixed number, and whole number. This includes recognition that when you multiply reciprocal numbers you get 1.

DECIMALS

Place value and number names

"Say" word names for decimals (up to about four decimal places).

Identify place values of digits on both sides of the decimal point.

Comparison and rounding

Compare two decimals using <, >, = and ≠.

Round decimals to the nearest specified place value (and especially the largest place value on each side of the decimal point).

Equivalent decimals and fractions

Express common fractions and mixed numbers with denominators of 10, 100, 1000 as decimals (and vice versa).

Express common fractions as decimals. Divide the numerator of a fraction by the denominator if necessary. [Use division, adding 0's to the dividend, to determine what fraction a smaller number is of a larger number.]

Multiply or divide a decimal by 10, 100, 1000, etc., to "move" its decimal point a certain number of places to the right or left.

Identify terminating, repeating, and non-repeating decimals.

Computation

Add, subtract, multiply, and divide decimals (or decimals and whole numbers). Add enough zeros to the dividend in division to "get" a certain number of decimal places in the quotient. (This usually means adding one more zero than the number of decimal places you need and then rounding back.)

PERCENT

Percent notation

Use percent forms, decimals, and basic fractions (e.g., $\frac{2}{10}$ or $\frac{35}{100}$) interchangeably, especially in mathematical expressions such as 30% of 60 = 18, $\frac{3}{10}$ of 60 = 18 and .3 x 60 = 18. Change any common fraction or decimal (e.g., 3/8 or .3927) to percent form including numbers larger than 1 (e.g., $1\frac{3}{4}$ and 1.067).

Use percent expressions involving fractions and decimals (e.g., $33\frac{1}{3}$% or 12.5%) as equivalents for expressions like 1/3 and 0.125. Round repeating decimals such as .6666 to a percent (e.g., 67% or, more typically, 66.7%).

Basic percent problems

Find a percent of a number (30% of 50 is ___) [recognize that 30% of 50 and .30 x 50 are equivalent expressions].

Find a number when a percent of it is known (15 is 30% of ___).

Find what percent one number is of another (15 is ___% of 50).

Applications

Determine solutions to applications of basic percent problems including sales tax and total price, amount of discount, sale price, amount and percent of markup, simple interest, and use of tables for compound interest.

[Substitute appropriate values into formulas (i.e., interpret formulas).]

RATIO AND PROPORTION

Equivalence of ratio

Determine whether two ratios are equivalent, and define a proportion as two equivalent ratios.

Solve a proportion

Solve a proportion, especially using algebra (e.g., $\frac{3}{6} = \frac{n}{72}$ so $6n = 216$ and $n =$ ____).

Applications

Determine equivalent ratios unknown values in proportions involving similar triangles (ratio of sides is given, or determined, and unknown side is found), scale drawings (ratio is given, true lengths or distances must be determined), rates (e.g., mileage, speed), and unit prices.

PROBABILITY AND STATISTICS

Listing Outcomes

List all possible outcomes of an experiment. Identify events that are possible and events that are not possible. Identify favorable outcomes that represent an event including favorable outcomes for "combined" events ("Event A or Event B," "Event A and Event B").

Use multiplication to find the total number of outcomes or the number of favorable outcomes. Here students are multiplying together the number of ways different parts of an outcome can occur rather than exhaustively listing outcomes (e.g., number of ways you can draw a first card x number of ways you can draw a second card--with or without replacement).

Determining probabilities

Determine probabilities for different events or combinations of events in an experiment where it is possible to list all possible outcomes and all favorable outcomes. (Combinations include "and," "or," and conditional events, and the complement of an event.) Determine probabilities where it is possible to calculate the number of possible outcomes and the number of favorable ones (even though there may be too many to list) using the arithmetic of combinations and factorials. Use 0 and 1 as probabilities for impossible events and certain events.

Determine probabilities for combined events that are mutually exclusive by adding probabilities that are given for the individual events. Multiply appropriate probabilities for combined

PROBABILITY AND STATISTICS (continued)

Determining probabilities (continued)

events when these events are independent (usually sampling with replacement) and when they are not independent (usually sampling without replacement). Determine the probabilities for the complement of an event.

Estimate probabilities from tables showing the empirical frequencies of occurrence of various outcomes and events.

Statistics

Measures of central tendency and dispersion

Identify mean median and mode for a set of values.

Identify the range of a set of values and the average deviation (absolute amounts) of observed values from their mean.

Graphs and tables

Answer questions about data and relationships among data shown in common forms of graphs including: pictographs, bar graphs, line graphs, circle graphs, and (occasionally) two bar or line graphs shown on the same axes. Answer questions about information shown in histograms and scattergrams including the idea of "fitting" a line to data in a scattergram (Mainly, it's a case of deciding whether the variables have a positive, negative, or zero relationship).

Identify from its description whether a set of data represents a statistical sample or a census (entire population).

INTEGERS

Positi and negative numbers

Identify positions for positive and negative numbers on the number line. Use positive and negative numbers to represent quantities in situations of increases and decreases (or gains and losses) where values above and below zero are possible. Identify the opposite number of an integer or the reason two integers are opposites (when you add them you get zero). Identify the absolute value of an integer. (Usually student practice doesn't include symbolic expressions like $|+3|$, yet.)

Compare integers using words like "more" or "less" and the symbols < and >.

Computation and rules for signs

Add, subtract, multiply and divide two integers, with and without some model such as the numberline as a guide. Identify the inverse of expressions involving addition and subtraction (e.g., adding +3 is the same as subtracting -3).

Add, subtract, multiply and divide rational numbers where rules for computing with integers are applied almost directly to expressions like $-\frac{3}{4}$, $+(\frac{3}{4})$, and -1.567. [Usually, students are not taught to interpret fractions with signs on both numerator and denominator (e.g., $\frac{-4}{-5}$ or $\frac{+2}{-3}$) until they have a formal course in algebra].

Graphing

Identify coordinates for points on a grid with two complete axes and four quadrants.

ALGEBRA

The work in algebra is mainly of two kinds: simplifying expressions using rules for order of operations and the distributive property ($3x + 2x = 5x$); and solving for "x" or for "x" and "y" in simple first order equations. Technically, the coefficients for "x" and "y" in these equations can be any rational number (e.g., -4, $\frac{3}{2}$, $+1.23$, $\frac{-4}{5}$, etc.) and so can the solutions. However, the general practice in textbooks through grade 8 is to restrict most of the example problems and problems intended for independent practice to use of whole numbers.

Numerical expressions and order of operations

Simplify numerical expressions like $3(47 + 5) + 4.5$ by applying standard rules for order of operations.

Expressions with variables

Substitute values for variables in an expression [when $x = 4$ $3x + 2 = 14$].

Identify equivalent expressions ($3x + 2x + 18$ and $5x + 18$).

Solving equations with one variable

Find a solution to a first order equation with one variable (e.g., $3x + 6 = 18$).

Generate solution sets for simple linear equations (e.g., $y = 2x + 18$). This usually means generating a table of values for x and y over a small set of five to ten possible values for x.

ALGEBRA (continued)

Solving equations with one variable (continued)

Generate solution sets for very simple first order inequalities (where "x" is isolated) such as $-5 < x < 15$. Usually this means listing some values that x can take and some values that it can't take.

Simultaneous solution of linear equations

Find a solution to two simple linear equations such as $y + 2x = 18$ and $x-y = 5$.

Graphs of equations

Plot points and sketch in lines for a few values of "x" and "y" in simple linear equations. Graphing may also be extended to include other basic types of equations such as $y = x^2 + 1$ and $y = \frac{1}{x}$ as long as equations are not too complicated (e.g., $y = 3x^2 + 15$ and $y = \frac{3}{2x}$ are probably within the bounds of regular 7th and 8th grade mathematics but textbooks will not likely have covered more complicated forms like $y = \frac{3}{x^2}$ and especially forms like $3y + 2x = 33$, and $y^2 + x^2 = 18$.

Writing equations

Write equations to fit simple word expressions such as "there are three times as many girls as boys," or "there are half as many red balls as blue ones."

ALGEBRA (continued)

Other topics

Solve two linear equations by graphing them and identifying where the graphs intersect. (Problems must be very restricted as to possible values for "x" and "y.")

Graph a first order inequality like $-4 < x > 12$ where "x" is isolated.

GEOMETRY

Basic concepts

Identify circle, square, triangle, rectangle, parallelogram, point, line, line segment, ray, plane, center of a circle, radius, diameter.

Perimeter and circumference

Find the perimeter of a polygon by adding measures of each side or by using a formula.

Determine the circumference of a circle by using the formula $c = \pi d$ or $c = 2\pi w$.

Area, surface area, and volume

Use standard formulas to determine (1) area of a triangle, rectangle, parallelogram, trapezoid and circle; (2) surface area of a rectangular prism, pyramid, cylinder, cone, or sphere; (3) volume of a rectangular prism, cylinder, pyramid, cone, or sphere.

Similar triangles

Know that similar triangles have corresponding sides whose lengths have the same ratio. In practice, students mainly have an opportunity to determine whether two triangles are similar by checking the ratios of their sides.

Determine an unknown side in two similar triangles where the ratio of the sides is given. Sometimes this is done by a formal process of setting up a proportion and solving for the missing value using techniques of algebra. Sometimes the approach is a more intuitive one where students are finding either the numerator or denominator of an equivalent fraction.

GEOMETRY (continued)

Congruence

Know that two figures are congruent if their corresponding parts are congruent. In independent practice this usually means that students determine whether two figures (usually triangles) are congruent by checking the congruence of corresponding sides and angles.

Know that there are three basic tests you can use to check the congruence of two triangles when you can't check all of the corresponding sides and angles. In practice, students will apply these tests to see whether two triangles have:

(a) all three sides are congruent

(b) two sides and the angle they include are congruent

(c) two angles and the side they include are congruent

Sometimes they tell "why" two triangles are congruent by designating one of these three principles usually as SSS, SAS, and ASA.

Know that parallel lines cut by a transversal form various pairs of congruent angles. In practice, students usually do the simpler tasks of identifying which angles are congruent in this kind of figure.

Pythagorean theorem and trigonometry

Use the Pythagorean theorem to determine the unknown length of one side of a right triangle. (Problems usually involve perfect squares.)

Use tables for sine, cosine, and tangent ratios to determine the unknown length of one side of a triangle (students look up values from a table).

GEOMETRY (continued)

Names of special angles and polygons

Give names for standard (regular) polygons (hexagon, octagon, etc.), triangles (isoceles, equilateral, etc.), and angles (acute, obtuse, straight, etc.).

Identify two angles that are supplementary or complementary.

Parallel and perpendicular relationships

Identify lines that are parallel or perpendicular. Identify planes that are parallel or perpendicular.

Symmetry and transformations

Identify figures that have line symmetry, radial (point) symmetry, or plane symmetry (solid figures).

Identify reflections, rotations, and translations of a figure on a two-dimensional grid.

Other Topics

Special relationships

Recall (or at least verify) special relationships like the following:

(1) the diameter of a circle is twice as long as the radius

(2) the sum of angles in a triangle is 180°

(3) the sum of angles in a polygon with n sides is (n-2) x 180° (this relationship can be verified by partitioning the interior of any polygon into non-overlapping triangles whose vertexes are the same as the vertexes of the polygon

(4) inscribed angles in a circle are half as big as the corresponding central angle

GEOMETRY (continued)

Special relationships (continued)

(5) a parallelogram is formed by connecting the midpoints of each side of a trapezoid

(6) the central angle in a regular polygon is congruent to the exterior angle formed by extending one of its sides

(7) the three perpendiculars drawn to each side of a triangle from an inside point have a sum that is equal to half of the triangle's height

(8) a line segment connecting the midpoints of two sides of a triangle is half as long as the third side

Arc length and area of a sector

Use a protractor to determine the length of an arc on a circle. Use arc length to determine the area of a sector of a circle formed with the arc.

Constructions

Follow directions for using (or simply use) a compass or straight edge to:

(1) bisect a line segment

(2) bisect an angle

(3) copy an angle

(4) copy a triangle

(5) construct one line perpendicular to another

(6) construct one line parallel to another

(7) inscribe a polygon inside a circle

GEOMETRY (continued)

Geometric transformations

Identify coordinates for the following figures in four quadrants

reflected figures

rotated figures

translated figures

magnified (similar) figures

MEASUREMENT

Length

Use a ruler to measure length to the nearest millimeter or the nearest 1/16 of an inch.

Weight, liquid capacity, temperature, time

Read values from the scale of a measurement device (weighing scale or balance, measuring container, thermometer, clock).

Angles

Use a protractor to measure an angle in degrees.

Money

Identify denominations of U.S. coins and bills and the total amount of money represented by a collection of coins and bills. Identify the smallest number and denominations of coins and bills that can be used to represent some amount of money.

Identify the total amount of a purchase and the amount of change one would receive from a payment.

Expressing measures and equivalent measures

Use decimals to express measures (especially when metric units are used).

Identify the precision and error in a measurement expression.

Determine equivalent metric units and equivalent customary units (e.g., 3.3 kg = ___ g). Usually students are not expected to recall all of the equivalent units. Most often, they get the basic information from a table.

MEASUREMENT (continued)

Expressing measures and equivalent measures (continued)

Perform computations on measures.

$$\begin{array}{r} 3 \text{ kg} \quad 4 \text{ g} \\ \underline{+\ 7 \text{ kg } 500 \text{ g}} \end{array}$$

including addition, subtraction, multiplication, and division.

Estimation

Make estimations of length, weight, liquid capacity, and volume (cubic units), especially for measures involving large units.

Identify the most reasonable or sensible unit for a particular kind of measurement.

Other topics

Identify fixed temperatures for freezing and boiling points of water and normal body heat.

SPECIAL TOPICS

Prime factorization

List the prime factors in a whole number. Usually confined to numbers less than about 100 or 150. Use prime factors to identify the greatest common divisor for the two numbers and the smallest common number that two given numbers will divide (this is also the lowest common denominator for two fractions).

Divisibility

Recall and use rules for divisibility by numbers from 2 to 9.

Scientific notation

Express whole numbers and decimals in scientific notation. [By the end of grade 8, most textbooks do not cover use of negative exponents (for numbers between 0 and 1), but some do.] Compare two numbers written in scientific notation, especially using the symbols <, >, and =.

Identify significant digits in a number expressed in scientific notation.

Compute with numbers in sciencific notation. (Use of the distributive law and generalizations for adding and subtracting exponents.).

Square root

Determine the squares of whole numbers (less than about 20).

Determine the square root of numbers that are perfect squares (4, 9, 16, 25, 36, 49, etc.).

SPECIAL TOPICS (continued)

Square root (continued)

Approximate the square root of whole numbers (up to about 200) using some process (there are several) of repeated division.

Relations and functions

Extend number sequences (including Pascal's triangle).

Generate number sequences according to some rule.

Generate tables showing input and output values for a function rule, especially for linear functions.

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